Heterogeneous emissions: climate risk mitigation and endogenous metrics

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The paper at a glance

- Different greenhouse gases have heterogeneous impacts on global temperature. For regulatory purposes, GHG emissions are expressed in CO-equivalent units, based on their Global Warming Potential (GWP).
- Climate mitigation model.
  - The policymaker and firms interact in a Stackelberg game.
  - Regulated firms emit different GreenHouse Gases. To be compliant, they suffer abatement costs and participate to dedicated emission reduction markets.
  - The policymaker seeks to minimize social costs and meet a temperature target, accounting for the distinct properties of each GHG.
- Result. An

  endogenous version of the Global Warming Potential (GWP) metric,

  which incorporates not only physical characteristics of the gases but also firms' abatement costs.

Introduction

Mechanisms for emission reduction

- Price-based regulation (pure emissions tax).
  The regulator fixes an explicit price per ton of CO_2 (e.g. €80/tCO_2); total emissions adjust endogenously.

  Where? Singapore, South Africa.
- Quantity-based regulation (cap-and-trade).***
  The regulator fixes an aggregate emissions cap; the carbon price is determined by permit trading on a so-called carbon market.

  Where? EU ETS, China ETS, South Korea, California, New Zealand.
- Hybrid systems (tax + cap-and-trade).
  Both price-based and quantity-based instruments coexist, typically with sectoral or partial overlap.

  Where? Sweden, Norway, Switzerland, Japan, Canada, UK.

What is a carbon/GreenHouse Gases reduction market?

- Carbon markets are financial mechanisms that encourage polluters to reduce their carbon footprint by trading carbon permits.
- Emissions reporting and regulatory oversight occur annually. Within an ETS firms' realized emissions minus the number of surrendered permits must be equal or less than a given threshold L. On the excess, they pay a penalty of €100:

$$\text{Cap} \; = 100 * (\text{Emissions - Permits -L})^{+}$$

- Firms are heterogeneous. Those with a deficit buy permits on the open market on dedicated exchanges (ICE, EEX) and firms that are virtuous on the regulated period sell.
- Current carbon permits compensate GHG emissions in carbon equivalence terms. Each permit covers one metric ton of CO2 equivalent, CO_2e.

EU ETS historical EUA prices chart

Climate risk management: emissions reporting

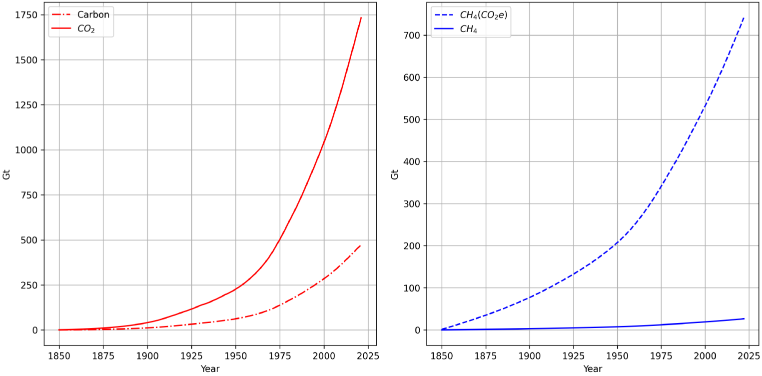
- Climate risk mitigation and related policies. Quantifying GHG footprints and developing strategies for their abatement to limit global temperature rises.
- Reporting emissions is key! It currently relies on the concept of CO_2 equivalence of the various GHGs, not on their native units.
- The equivalence coefficient for an horizon $\tilde{T}$ of a gas is denoted by $GWP_{\tilde{T}}$. For gas j, it quantifies time-integrated radiative forcing of pulse emission of one tonne over the horizon $\tilde{T}$, relative to the impact of one-tonne pulse emissions of CO_2:

$$\mathrm{GWP}^j_{\tilde{T}} = \frac{\int_0^{\tilde{T}} RF_j(t)\,dt}{\int_0^{\tilde{T}} RF_{\mathrm{CO}_2}(t)\,dt}$$

in which RF is the respective radiative efficiency function.

Cumulative emissions of carbon dioxide and methane

Figure: Cumulative emissions of CO_2 and CH_4 to date. Source: Jones et al., Scientific Data 2023.



Carbon accounting: not so stringent

- Examples of GWP(CH_4) according to the IPCC:

IPCC Report	GWP_{20}	GWP_{100}
SAR	56	21
TAR	62	23
AR4	72	25
AR5	84	28
AR6	[79.7, 81.1, 82.5]	[27.0, 28.4, 29.8]

- GHG Protocol: "Companies: shall use 100-year GWP values from the The Intergovernmental Panel on Climate Change; should use GWP values from the most recent Assessment Report, but may choose to use other IPCC Assessment Reports", etc.

- CO_2e bases differ among jurisdictions/initiatives: the Global Methane Pledge, Climate Leadership and Community Protection Act (CLCPA) of New York in 2019, the Climate Solutions Now Act of Maryland in 2022, etc. all require 20-year GWPs.

Trading GHGs against each other

- One carbon permit is retired for one metric tonne of CO_2e. That is the price per ton of gas j is:

$$P_j = \text{GWP}^j_{\tilde{T}} * P_{CO_2}.$$

- Example. On the AR6-GWP_{100} CO_2e basis, 1 **ton** $\mathbf{CH_4}$ $= 29.8$ **ton** $\mathbf{CO_2}$**e.** Reinterpreted, abating 1 tonne of methane saves us to 29.8 carbon permits that we can surrender for other emissions.

- **Question 1.** How to determine specific GHG **abatement incentives**?
- Sector-specific GHG abatement costs are vastly different.
  Methane can sometimes abated very cheaply: 1 flared tonne produces 2.75 tonnes of CO_2. Feature not captured by the GWP.
- What is the net impact on the temperature of cheaply abating 1 tonne of CH_4, a short lived gas, while sparing $27.05 = 29.8 - 2.75$ carbon permits?

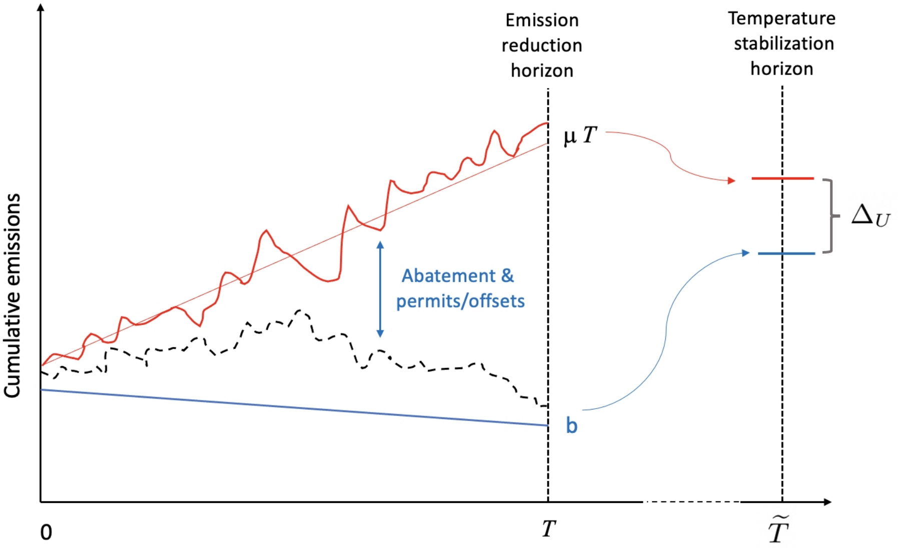
The horizon problem

- **Question 2.** Is the current penalization of methane, compared to CO_2, truly optimal for mitigating climate change?
- Answer. It depends on the regulatory target horizon $\tilde{T}$.
- Methane is a short lived gas, lifetime 12.4 years. Its penalization via its high GWP on shorter term horizons (20, 100 years) aligns with more myopic Climate Mitigation measures.
- A GHG that aligns with shorter-term temperature reduction targets may conflict with longer-term Climate Risk Mitigation objectives, due to a slower but more persistent impact.
- In case the policymaker cares about future generations, they should penalize more long-lived gasses such as CO_2, Sulfur Hexafluoride SF_6.

What do we do here?

- We adopt a framework similar to B. and Aïd, Math Finance 2023 and B., SIFIN 2025. Flexible enough!
- Regulator and Firms playing a Leader-Follower game in a Linear-Quadratic setup.
- Firms from sector j emit only one type of gas, GHG_j, and they trade permits in the market dedicated to their GHG emissions. Firms then minimize their abatement and GHG_j trading costs so to respect a regulator's given policy (targets). Imposing market clearance, we obtain the permits price P_j of the permits per sector.
- The regulator minimizes over the targets the equilibrium social costs of compliance, imposing an extra constraint on the temperature at a (possibly very far) horizon $\tilde{T}$.
- The solution(s) highlight the contribution of the various gases to the temperature impact. This encompasses average abatement costs and radiative forcing of each gas, reflected on the optimal induced prices P_j^*. Endogenous equivalences between gases alternative to the GWP.

In a nutshell...



Setup

The economy

- K emissive sectors.
- Firm ij is company i among the N_j firms operating in sector $j \in \{1, \cdots, K\}$.
- $(\Omega, \mathbb{F}, \mathbb{P})$, with $\mathbb{F} := (\mathcal{F}_t)_{t \geq 0}$ augmented natural filtration of an

$$N = N_1 + \ldots + N_K + K + 1$$

  dimensional Brownian motion.
- Each firm faces quadratic abatement costs and is subject to a regulatory target b_{ij} on cumulative (net) emissions at the end of time horizon $[0, T]$.
- K markets of permits.

Emissions and net position

- Firm ij's controlled emission dynamics E^a_{ij}, in native units, are given by

$$dE^a_{ij}(t) = (\mu_{ij} - a_{ij}(t))dt + \sigma_{ij}(t)dW_{ij}(t),$$

  a_{ij} is the abatement rate.

- Firm and sector specific shocks, as well as aggregate shocks:

$$W_{ij}(t) = \sqrt{1 - \left(h^{ij}_0\right)^2 - \left(h^{ij}_j\right)^2}\,\widetilde{W}_{ij}(t) + h^{ij}_j \widetilde{W}_j(t) + h^{ij}_0 \widetilde{W}_0(t)$$

- To be compliant, firms can abate at at rate a_{ij} or trade permits at rate θ_{ij} - square integrable - to offset emissions at the terminal date T.

The single firm's problem, horizon T

- Firm ij solves their standalone cost minimization under given regulatory constraints over abatement and trade:

$$\begin{cases} \inf_{a_{ij},\theta_{ij}\in\mathcal{A}} J^{ij}(a_{ij},\theta_{ij} \mid b_{ij},\varepsilon) \\ \mathbb{E}\left[\left(E^a_{ij}(T) - \int_0^T \theta_{ij}(t)\,dt - b_{ij}\right)^2\right] \le \varepsilon^2 \end{cases}$$

- The objective function is:

$$J^{ij}(a_{ij},\theta_{ij} \mid b_{ij},\varepsilon) := \mathbb{E}\left[\int_0^T h_{ij}a_{ij}(t) + \frac{1}{2\eta_{ij}}(a_{ij}(t))^2 dt + \int_0^T \theta_{ij}(t)P_j(t)dt\right],$$

in which $h_{ij},\eta_{ij} > 0$ are specific abatement cost parameters, P_j the at the moment exogenous martingale market price of permits and:

  - the GHG emission target cumulated level b_{ij} (possibly negative),
  - the tolerance level $\varepsilon > 0$.

are the regulator's controls.

Assumption SQV

- Slow Quadratic Variation (SQV) on the emissions:

$$\mathbb{E}\left[\int_0^T \frac{(\sigma_{ij}(t))^2}{T-t}dt\right] < \infty,$$

- Why? LQ degenerate problem.
- Not covered by Bismut, SICON 1976 or in the subsequent (huge) literature on LQ problems.
- Bank, Soner, Voß MaFe 2017 and B. and Žitković, Bernoulli 2024:

  Slow Quadratic Variation is necessary and sufficient to recover optimizers.
- In the single firm's cost minimization: the exogenous P_j is also required to satisfy SQV. But at equilibrium, the endogenous equilibrium price $\hat{P}_j$ will be automatically SQV.

The single firm's problem solution

- Lagrange multipliers method.
- Strict convexity in abatement implies that $\widetilde{a}_{ij}$ is unique and given by:

$$\widetilde{a}_{ij}(t) = \eta_{ij}(P_j(t) - h_{ij}). \tag{1}$$

- The optimal trading strategy $\widetilde{\theta}_{ij}$ is not unique. An optimal strategy can be written as follows:

$$\begin{aligned} \widetilde{\theta}_{ij}(t) &= \eta_{ij} h_{ij} + \mu_{ij} - \frac{\frac{1+2\widetilde{y}_j \eta_{ij} T}{2\widetilde{y}_j} P_j(0) + b_{ij}}{T} + \\ &+ \int_0^t \frac{1}{T-s} \left(-\frac{1+2\widetilde{y}_j \eta_{ij}(T-s)}{2\widetilde{y}_j} dP_j(s) + \sigma_{ij}(s) dW_{ij}(s) \right). \end{aligned} \tag{2}$$

- Note. The optimal multiplier $\widetilde{y}_j$ is constant across firms in Sector j:

$$\widetilde{y}_j = \frac{\sqrt{\mathbb{E}[P_j^2(T)]}}{2\varepsilon} = \frac{\sqrt{E[\langle P_j \rangle(T)] + P_j^2(0)}}{2\varepsilon}. \tag{3}$$

The optimal multiplier is an endogenous cap

- The problem is equivalent to:

$$\inf \mathbb{E}\left[\int_0^T h_{ij} a_{ij}(t) + \frac{1}{2\eta_{ij}}(a_{ij}(t))^2 dt + \int_0^T \theta_{ij}(t) P_j(t) dt + \right.$$
$$\left. \widetilde{\mathbf{y}}_{\mathbf{j}} \left(\left(E^a_{ij}(T) - \int_0^T \theta_{ij}(t)\, dt - b_{ij} \right)^2 - \epsilon^2 \right) \right].$$

- Direct interpretation of the multiplier as the optimal, endogenous cap.

Equilibrium within the single sector, assumptions

- Define average target per sector $\bar{b}_j$, average inflexibility $\bar{\eta}_j$ and average mixed cost $\bar{c}_j = \frac{\sum_i h_{ij}\eta_{ij}}{N_j}$.
- Assumption 1. For all sectors, the average target per sector $\bar{b}_j$ verifies:

$$\bar{b}_j \leq (1-\delta_j)\left(\overline{\mu}_j + \overline{c}_j\right) T, \text{ for some } \delta_j \in (0,1). \tag{4}$$

- Assumption 2. The regulator's tolerance level ε belongs to the interval below:

$$\varepsilon \in \left(0, \min_j \frac{1}{N_j}\left(\int_0^T \mathbb{E}\left[(\sigma_j(t))'\Lambda_j\sigma_j(t)\right] dt\right)^{\frac{1}{2}}\right), \tag{5}$$

in which Λ_j is the covariance matrix of the sector j-th noise vector $(W_{ij})_{i=1}^{N_j}$.

Equilibrium in sector j

- For given $\mathbf{b}_j = (b_{ij})_{i=1,\dots,N_j}$ and ε satisfying Assumptions 1, 2, the equilibrium permit price $\widehat{P}_j$ is unique:

$$d\widehat{P}_j(t) = \frac{2\widehat{y}_j}{1+2\widehat{y}_j\overline{\eta}_j(T-t)}\, d\overline{W}_j(t), \quad \widehat{P}_j(0) = \frac{2\widehat{y}_j((\overline{\mu}_j+\overline{c}_j)T-\overline{b}_j)}{1+2\widehat{y}_j\overline{\eta}_j T} > 0.$$

  How? impose market clearing $\sum_i \tilde{\theta}_{ij}(t) = 0$ and derive the SDE for P_j.
- The optimal multiplier $\widehat{y}_j = \widehat{y}_j(\varepsilon, \overline{b}_j)$ is the solution to:

$$\epsilon^2 = \left(\frac{((\overline{\mu}_j+\overline{c}_j)T-\overline{b}_j)}{1+2y_j\overline{\eta}_j T}\right)^2 + \int_0^T \frac{1}{(1+2y_j\overline{\eta}_j(T-t))^2}\mathbb{E}\left[(\sigma_j(t))'\Lambda_j\sigma_j(t)\right] dt.$$

- Each firm ij's optimal $\widehat{a}_{ij}$ is unique and given by inserting $\widehat{P}_j$ into the standalone $\widetilde{a}_{ij}$:

$$\widehat{a}_{ij}(t) = \eta_{ij}(\widehat{P}_j(t) - h_{ij}).$$

- One optimal trading strategy $\widehat{\theta}_{ij}$ is obtained by substituting $\widehat{P}_j$, $\widehat{y}_j$ in the standalone $\widetilde{\theta}_{ij}$.

Optimal GHG prices: CO_2 and CH_4

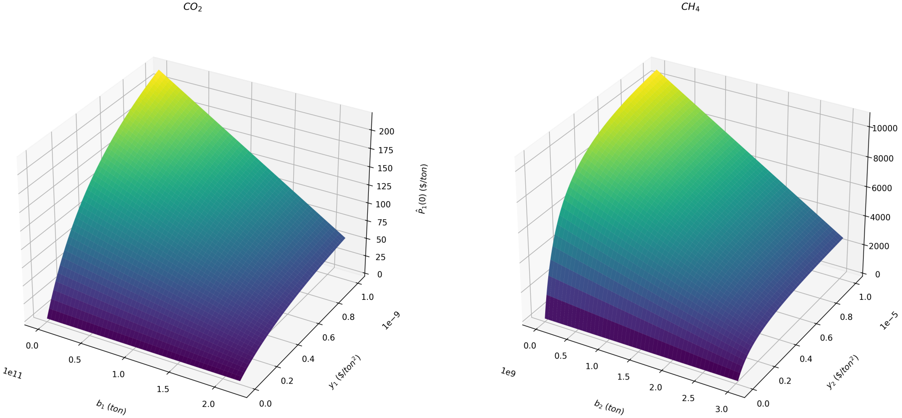


Figure: Optimal emission prices for CO_2 and CH_4 permits for different emission targets and $y_j = y_j(\varepsilon)$ for 2015 abatement cost parameters h and η.

Climate mitigation

Firms' pollution impact on the temperature

- Focus on the impact on one layer only of the atmosphere, the so-called the mixed layer. It is the lowest layer of the troposphere.
- Influence of the human activities on the temperature of the mixed layer at a future date $\tilde{T}$. Emissions cease after $T \leq \tilde{T}$.
- We define a proxy for the mixed layer temperature, an index $I^{\mathbf{a}}$:

$$\begin{cases} dI^{\mathbf{a}}(t) = -\alpha I^{\mathbf{a}}(t)dt + \frac{1}{C} dR^{\mathbf{a}}(t) \\ I^{\mathbf{a}}(0) = 0, \end{cases}$$

  in which the constants C, α denote capacity/absorption characteristics of the layer and the exogenous $R^{\mathbf{a}}$ is the forcing due to the gases concentrations (i.e. emissions).
- Assumption radiative forcing is linear in the specific gas emissions, and also across different gases. This approximation is good enough for the study (Pierrehumbert, Annual review of earth and planetary sciences, 2014).

Explicit formula and concrete cases

- Solution:

$$I^{\mathbf{a}}(t) = \sum_j \sum_{ij} \frac{1}{C} \int_0^t ds\, e^{\alpha(s-t)} \int_0^{T\wedge s} f_j(v-s) dE^a_{ij}(v).$$

- In the above, f_j is the radiative efficiency of gas j.
- Examples are:
  - For CH_4, exponential decay:

$$f_j(v) = d_j e^{\beta_j v}, v \in (-\infty, 0]$$

    with $\beta_j = \frac{1}{12.4}$, $d_j = 0.565$.
  - For CO_2, mixture of exponentials and eternal component:

$$f_j(v) = d_{j,0} + \sum_{k=1}^{3} d_{j,k}\, e^{\beta_{j,k} v}.$$

    From Pierrehumbert 2014:
    $f_j(v) = 0.004223(0.27218 e^{v/8.696} + 0.14621 e^{v/93.3} + 0.13639 e^{v/645.87} + 0.44522)$.

The Regulator's problem

- In addition to the emission target reduction at time T, the regulator also adopts a constraint on the global warming induced over a longer horizon $\tilde{T}$, to take into account the legacy for future generations.

$$(\mathcal{R}) \quad \begin{cases} \inf_{\mathbf{b}\in\mathcal{B}} \sum_{j=1}^{K} \sum_{i=1}^{N^j} J^{ij}\left(\hat{a}_{ij}, \hat{\theta}_{ij} \mid b_{ij}, \varepsilon\right) \\ \mathbb{E}\left[I^0(\widetilde{T}) - I^{\hat{a}}(\widetilde{T})\right] = \Delta U \end{cases}$$

- The expectation in the constraint is easily computed:

$$\mathbb{E}\left[I^0(\widetilde{T}) - I^{\hat{a}}(\widetilde{T})\right] = \sum_{j=1}^{K} N_j \overline{a}_j(0) \int_0^{\tilde{T}} dt\, e^{\alpha(t-\widetilde{T})} \int_0^{T\wedge t} ds\, f_j(s-t)$$

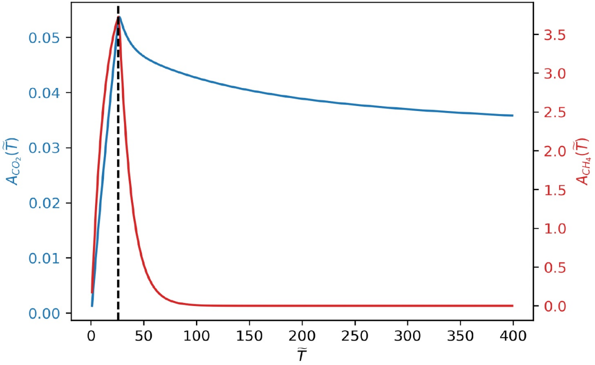
so that the constraint becomes:

$$\sum_{j=1}^{K} N_j(\overline{\eta}_j \hat{P}_j(0) - \bar{c}_j) \int_0^{\tilde{T}} dt\, e^{\alpha(t-\widetilde{T})} \int_0^{T\wedge t} ds\, f_j(s-t) = \Delta U$$

- Numerical solution.

The annuity factors

- The factor $A_j(T, \tilde{T}) := \int_0^{\tilde{T}} dt\, e^{\alpha(t-\widetilde{T})} \int_0^{T\wedge t} ds\, f_j(s-t)$ quantifies the impact of a unitary flow of GHG j in $[0, T]$ at the horizon $\tilde{T}$.



Comparing CO_2's and methane's global warming **annuity factors** $A(\widetilde{T})$ over different time horizons; $T = 2050$. Source: Biagini, Biffis and Salezadeh-Nobari (2025).

A single GHG: $K = 1$

- The optimally regulated equilibrium price is

$$P_1^*(0) = \frac{1}{\overline{\eta}_1}\left(\frac{\Delta U}{N_1 A_1(\widetilde{T})} + \overline{c}_1\right) =: p_1^*(0)$$

  The physical characteristics of the gas at hand, as well as its abatement costs, characterize the equilibrium price of the emission permit!

- The quantity $\Delta U / N_1 A_1(\widetilde{T})$ reveals how the temperature target is divided among firms and discounted by the relevant annuity factor to obtain the optimal abatement:

$$\overline{a}_1^*(0) = \overline{\eta}_1 P_1^*(0) - \overline{c}_1 = \frac{\Delta U}{N_1 A_1(\widetilde{T})}.$$

Two GHGs, $K = 2$

- GHG_1 captures a long-lived gas (CO_2) and GHG_2 a short-lived one (methane).
- One gas can be used as a benchmark: GHG_1, in analogy with the GWP principle. Assume $N_1 = N_2 = N$.
- In terms of abatement we get:

$$\bar{a}_2^*(0) = \frac{\Delta U}{N A_2(\widetilde{T})} - \frac{A_1(\widetilde{T})}{A_2(\widetilde{T})} \, \bar{a}_1^*(0).$$

- In terms of permits prices:

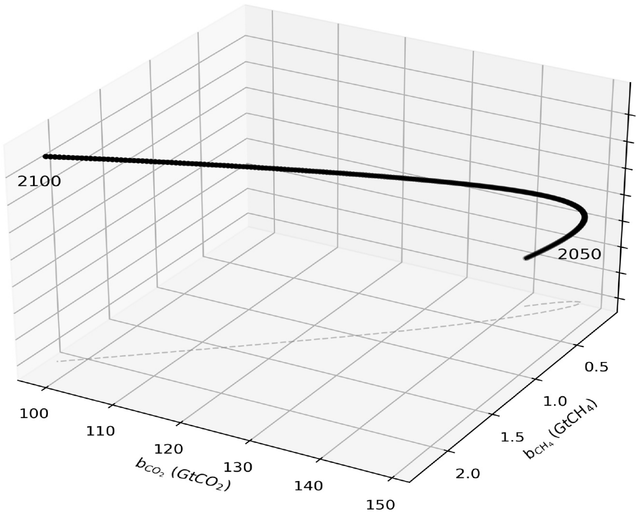
$$P_2^*(0) = p_2(0) - \frac{A_1(\widetilde{T})}{A_2(\widetilde{T})} \, \frac{\overbrace{(\bar{\eta}_1 P_1^*(0) - \bar{c}_1)}^{\bar{a}_1^*(0)}}{\bar{\eta}_2},$$

where $p_2(0)$ denotes the standalone emission permit price of GHG_2.

Effort sharing effect, GWP and abatement costs

- Prices are lower than in the absence of another GHG; this is because the effort to meet a temperature reduction target is now shared across two emission markets.
- Price reduction depends on the ratio between the GHG-specific annuity factors, which jointly capture differences in radiative forcings and persistency of the two GHGs (similarly to GWP).
- As GHG_1 represents here a long-lived gas, the price of the short-lived gas, P_2^*, decreases as the climate risk mitigation horizon $\widetilde{T}$ increases. This is in line with conversion factors based on GWP, with the important difference that the latter abstract away from GHG-abatement costs.

Model calibration: $(b^*_{CO_2}, b^*_{CH_4})$



Model calibration based on Jones et al. (2023) for emission dynamics, $\widetilde{T} = T$.

Perspectives

- Need for global harmonization of emission reporting standards removing optionality and divergence across jurisdictions.
- National/corporate/policy emission targets need to explicitly differentiate between GHGs and use native units of measure, to reduce vulnerability to greenwashing and carbon arbitrage.
- Environmental policy/economics and climate finance studies need to tackle sizeable measurement errors associated with heterogeneity and optionality affecting CO_2e bases, as well as parameterizations informing model calibrations.

Thank you all very much!