Extension of Strassen theorem for arbitrage-free prices of American options

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Problem formulation

Setting:

- Model-independent framework, discrete-time
- Deterministic numéraire B (increasing, $B_0 = 1$), risky asset X (discounted $\bar{X} = X/B$)
- We observe prices of liquidly traded options written on X

Question: Does there exist an arbitrage-free model compatible with market prices?

I.e. $(\Omega, (\mathcal{F}_t)_t, \mathbb{Q})$, X adapted s.t. $\bar{X}$ martingale and mk prices = risk-neutral eval. under $\mathbb{Q}$

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- For t-European options $\varphi_t(X_t)$: mk price $(\varphi_t(X_t)) = \mathbb{E}_{\mathbb{Q}}[\varphi_t(X_t)/B_t]$
- For t-American options $\Phi(X) = (\varphi_s(X_s))_{s=0,1,..,t}$:

$$\text{mk price } (\Phi(X)) = \mathbb{E}_{\mathbb{Q}}\left[\varphi_0(X_0) \vee \mathbb{E}_{\mathbb{Q}}\left[\frac{\varphi_1(X_1)}{B_1} \vee \ldots \vee \mathbb{E}_{\mathbb{Q}}\left[\frac{\varphi_t(X_t)}{B_t}\Big|\mathcal{F}_{t-1}\right] \ldots \Big|\mathcal{F}_1\right]\Big|\mathcal{F}_0\right]\right]$$

Models compatible with market prices

Recall: convex order $\mu \prec_{\text{cvx}} \nu$ means that $\mu(f) \equiv \int f d\mu \leq \int f d\nu \equiv \nu(f)$ $\forall$ convex f

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European t-Put $\forall k$	Breeden Litzenberger $\Longrightarrow$ ①	$\mathcal{L}_{\mathbb{Q}}(\bar{X}_t)$ $\forall$ compatible $\mathbb{Q}$	Strassen $\Longrightarrow$ ②	NA criteria:
$p_t^E(k)$ mk price		$p_t^E(k) = \mathbb{E}_{\mathbb{Q}}[(k/B_t - \bar{X}_t)_+]$		$\exists\, \mathbb{Q} \in \mathcal{M}(\mu_1, \mu_2)$
of $(k - X_t)_+$		$= \int (k/B_t - x)_+ \, \mu_t(dx)$		$\Leftrightarrow$ $\mu_1 \prec_{\text{cvx}} \mu_2$

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$\xrightarrow[\text{①}]{\text{Breeden Litzenberger}}$

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$p_t^E(k) = \mathbb{E}_{\mathbb{Q}}[(k/B_t - \bar{X}_t)_+]$
$= \int (k/B_t - x)_+ \, \mu_t(dx)$

$\xrightarrow[\text{②}]{\text{Strassen}}$

NA criteria:
$\exists\, \mathbb{Q} \in \mathcal{M}(\mu_1, \mu_2)$
$\Leftrightarrow$ $\mu_1 \prec_{\text{cvx}} \mu_2$

Goal: understand if something similar holds for American t-Puts prices:

American t-Put $\forall k$
$p_t^A(k)$ mk price

$\xrightarrow[\text{①}]{???}$

some measure ν_t

$\xrightarrow[\text{②}]{???}$

NA criteria:
$\nu_1 \prec_{???} \nu_2$

Potential functions

Definition (Potential function)

The potential function of $\mu \in \mathcal{P}_1(\mathbb{R})$ is given by: $p_\mu(k) = \int (k - x)_+ \mu(dx), \; k \in \mathbb{R}$.

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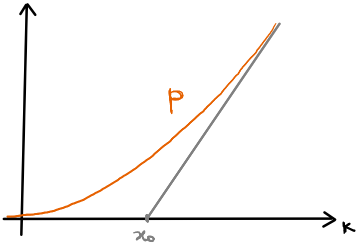
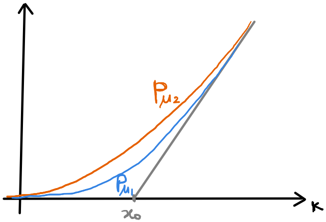
<u>Remark:</u> a function $p : \mathbb{R} \to \mathbb{R}$ is potential function of some $\mu \in \mathcal{P}_1(\mathbb{R}) \iff \exists\, x_0 \in \mathbb{R}$:
(i) p increasing, convex; (ii) $p(k) \geq (k-x_0)_+$; (iii) $\lim_{k\to-\infty} p(k) = 0$, $\lim_{k\to\infty} p(k) - (k-x_0)_+ = 0$



Potential functions and convex order

$$\mu_1 \prec_{cvx} \mu_2 \qquad \Longleftrightarrow$$



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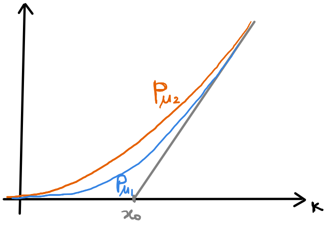
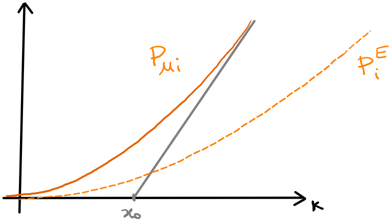


In particular: $p_i^E(k) = \int (k/B_i - x)_+ \, \mu_i(dx) \implies p_{\mu_i}(k) = \int (k-x)_+ \mu_i(dx) = p_i^E(B_i k)$



American options: first remarks

Note: NA-compatible American t-Puts prices $\left(p_t^A(k)\right)_{k\in\mathbb{R}}$ satisfy properties (i) – (iii) $\Rightarrow$ potential of some ν_t:

$$p_t^A(k) = \int (k-x)_+ \, \nu_t(dx), \quad k \in \mathbb{R}.$$

Therefore we have $''\underset{①}{\Longrightarrow}''$.

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Therefore we have $'' \underset{①}{\Longrightarrow} ''$.

Next: look for $'' \underset{②}{\Longleftarrow} ''$, i.e. NA criteria based on some order between the ν_t's.

Example in next slide shows that convex order is not enough.

American options: first remarks

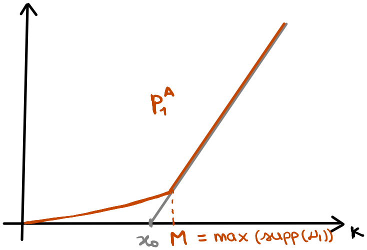
Remark

Let $\mathcal{F}_0$ trivial, $X_0 = x_0$. For the observed price curves $p_0^A(k) = (k - x_0)_+$ and p_1^A, TFAE:

- *$\exists$ arbitrage-free compatible model $(\Omega, (\mathcal{F}_t), \mathbb{Q}, \bar{X})$:*

$$p_1^A(k) = (k - x_0)_+ \vee \mathbb{E}_{\mathbb{Q}}[(k/B_1 - \bar{X}_1)_+]$$

- *(i) $\nu_0 <_{cvx} \nu_1$, where p_0^A potential of ν_0 (i.e. $\nu_0 = \delta_{x_0}$) and p_1^A potential of ν_1, and*
  (ii) $\nu_1(M) > 1 - 1/B_1$, where $M := \max \operatorname{supp}(\nu_1)$



Beyond convex order

Definition (β-biased measure and β-biased order)

Let $\beta \in [0,1), \mu, \nu \in \mathcal{P}_1(\mathbb{R})$ centered in x_0. We say that

- ν is *simple β-biased* if $\nu|_{[x_0,+\infty)}$ consists of a single atom with mass $\geq \beta$
- ν is *β-biased* if $\nu = \int \nu_u du$, ν_u simple β-biased probabilities
- $\mu \prec_\beta \nu$ if, for all continuous $g : \mathbb{R} \to \mathbb{R}$ s.t. $|g(x)| \leq C(1 + |x|)\ \forall x \in \mathbb{R}$ for some $C > 0$,

$$\mu(g_\beta) \leq \nu(g), \quad g_\beta(x) := \inf\left\{\int g\, d\rho : \rho \text{ s.t. } \int y\, d\rho = x,\ \rho(\{\max \operatorname{supp}(\rho)\}) \geq \beta\right\}$$

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Remarks:

- ν is β-biased $\iff$ $\delta_{x_0} <_\beta \nu$

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- ν is β-biased $\iff$ $\delta_{x_0} \prec_\beta \nu$
- For $\beta_2 > \beta_1$: $\prec_{\beta_2} \implies \prec_{\beta_1}$

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Remarks:

- ν is β-biased $\iff$ $\delta_{x_0} \prec_\beta \nu$
- For $\beta_2 > \beta_1$: $\prec_{\beta_2} \implies \prec_{\beta_1}$
- For $\beta = 0$: g_0 = cvx hull, and $\prec_0 \equiv \prec_{\text{cvx}}$
- For $\beta = 1$: $g_1 = g$, and $\mu \prec_1 \nu \iff \mu = \nu$

Biased Strassen theorem

Definition (β-biased martingale coupling)

Let $\beta \in [0,1), \mu, \nu \in \mathcal{P}_1(\mathbb{R})$ centered in x_0. The set of β-biased martingale couplings is

$$\mathcal{M}_\beta(\mu,\nu) := \left\{\pi \in \mathrm{Cpl}(\mu,\nu) \text{ s.t. } \delta_x \prec_\beta \pi_x \text{ for } \mu\text{-a.e. } x\right\} \subseteq \mathcal{M}(\mu,\nu)$$

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- $\pi \in \mathcal{M}_\beta(\mu,\nu) \iff \pi = \mathcal{L}(M_0, M_1)$ for some martingale M on some probab. space s.t. $\mathcal{L}(M_1|\mathcal{F}_0)$ has an atom of mass $\geq \beta$ at max of its support

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Theorem (Biased Strassen Theorem)

Let $\beta \in [0,1), \mu, \nu \in \mathcal{P}_1(\mathbb{R})$. Then

$$\mu <_\beta \nu \iff \mathcal{M}_\beta(\mu,\nu) \neq \emptyset$$

A stronger biased order

For NA criteria for prices of American options, we need something a bit stronger:

Definition (Strong β-biased order, strongly β-biased martingale coupling)

Let $\beta \in (0,1), \mu, \nu \in \mathcal{P}_1(\mathbb{R})$ centered in x_0. We say that

- ν is *strongly β-biased* ($\delta_{x_0} \prec_{s\beta} \nu$) if $\nu = \int \nu_u du$, ν_u simple β_u-biased, $\beta_u > \beta$
- $\mu \prec_{s\beta} \nu$ if $\exists\, \pi \in \mathcal{M}(\mu,\nu)$ s.t. $\delta_x \prec_{s\beta} \pi_x$ for μ-a.e. x, and we write $\pi \in \mathcal{M}_{s\beta}(\mu,\nu)$

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- In above toy market example: $\nu_0 = \delta_{x_0} \prec_{s\beta} \nu_1$, with $\beta = 1 - 1/B_1$

....spoiler: the order $\prec_{s\beta}$ will give us $''\underset{②}{\Longrightarrow}''$

Integral characterization of the convex order

Theorem

For $\mu, \nu \in \mathcal{P}_1(\mathbb{R})$, TFAE:

- $\mu \prec_{cvx} \nu$
- *Brownian martingale representation: $\exists\, M_0 \sim \mu$ and predictable $H \geq 0$ s.t.*

$$M_0 + \int_0^1 H_t \, dB_t \ \sim \nu$$

- *Poisson martingale representation: $\exists\, M_0 \sim \mu$ and predictable $H \geq 0$ s.t.*

$$M_0 + \int_0^\infty H_t \, d\widetilde{N}_t \ \sim \nu$$

B = Brownian motion, N = standard Poisson, $\widetilde{N}_t = t - N_t$ = − compensated Poisson

Integral characterization of the β-biased order

Theorem

Let $\mu, \nu \in \mathcal{P}_1(\mathbb{R})$ and $\beta \in (0,1)$. Then $\mu <_\beta \nu$ IFF $\exists\, M_0 \sim \mu$ and predictable $H \geq 0$ s.t.

$$M_0 + \int_0^{t_\beta} H_t \, d\widetilde{N}_t \sim \nu, \qquad t_\beta = ln(1/\beta)$$

Integral characterization of the β-biased order

Theorem

Let $\mu, \nu \in \mathcal{P}_1(\mathbb{R})$ *and* $\beta \in (0,1)$. *Then* $\mu <_\beta \nu$ *IFF* $\exists\, M_0 \sim \mu$ *and predictable* $H \geq 0$ *s.t.*

$$M_0 + \int_0^{t_\beta} H_t \, d\widetilde{N}_t \sim \nu, \qquad t_\beta = ln(1/\beta)$$

Example: H deterministic $\implies$ martingale $M_t := x_0 + \int_0^t H_s \, d\widetilde{N}_s \sim \nu_t$ has

$$\max \operatorname{supp}(\nu_t) = x_0 + \int_0^t H_s \, ds,$$

taken with positive probability $= \mathbb{P}(\tau_1 > t) = e^{-t}$, where τ_1 first jump time of N

$\implies \nu_t$ β-biased, with $\beta = e^{-t}$

Integral characterization of the strong β-biased order

Theorem

Let $\mu, \nu \in \mathcal{P}_1(\mathbb{R})$ and $\beta \in (0,1)$. Then $\mu \prec_{s\beta} \nu$ IFF $\exists\, M_0 \sim \mu$ and predictable $H \geq 0$ s.t.

$$M_0 + \int_0^{\tau_\beta} H_t \, d\widetilde{N}_t \sim \nu,$$

for some stopping time $\tau_\beta < ln(1/\beta)$

Note:

- $\mu \prec_{\beta+\varepsilon} \nu \implies \mu \prec_{s\beta} \nu$
- $\mu \prec_{s\beta} \nu \;\not\Longrightarrow\; \mu \prec_{\beta+\varepsilon} \nu$

Gluing property of the biased orders

Lemma

- *If* $\mu \prec_{\beta_1} \nu$ *and* $\nu \prec_{\beta_2} \eta \implies \mu \prec_{\beta_1\beta_2} \eta$
- *If one of them strong* $\implies \mu \prec_{s\beta_1\beta_2} \eta$

Illustration: consider the martingale $M_t := M_0 + \int_0^t H_s \, d\widetilde{N}_s \sim \nu_t$

For $s < t < u$: $\nu_s \prec_{\exp\{-(t-s)\}} \nu_t \prec_{\exp\{-(v-t)\}} \nu_u$

and $\nu_s \prec_{\exp\{-(u-s)\}} \nu_u$

Biased order and NA American prices

Theorem

For two maps $p_0^A, p_1^A : \mathbb{R} \to \mathbb{R}$, TFAE:

- $\exists\ (\Omega, (\mathcal{F}_t)_{t=0,1}, \mathbb{Q})$ *and a martingale* $(\bar{X}_0, \bar{X}_1)$ *on it such that*

$$p_0^A(k) = \mathbb{E}_{\mathbb{Q}}\left[(k - \bar{X}_0)_+\right], \quad k \in \mathbb{R}$$
$$p_1^A(k) = \mathbb{E}_{\mathbb{Q}}\left[(k - \bar{X}_0)_+ \vee \mathbb{E}_{\mathbb{Q}}[(k/B_1 - \bar{X}_1)_+ | \mathcal{F}_0]\right], \quad k \in \mathbb{R}$$

- p_i^A *potential function of* ν_i, $i = 0, 1$, *such that*

$$\nu_0 \prec_{s\beta_1} \nu_1, \qquad \text{where } \beta_1 = 1 - 1/B_1.$$

$\to$ Analogous for p_n^A curve of American n-Put prices, with $\nu_0 \prec_{s\beta_n} \nu_n$ for $\beta_n = 1 - 1/B_n$

Biased order and NA American prices

Theorem

For two maps $p_1^A, p_2^A : \mathbb{R} \to \mathbb{R}$, TFAE:

- $\exists\, (\Omega, (\mathcal{F}_t)_{t=0,1,2}, \mathbb{Q})$, $\mathcal{F}_0$ *trivial, and martingale* $(x_0, \bar{X}_0, \bar{X}_1)$ *on it such that*

$$p_1^A(k) = (k - x_0)_+ \vee \mathbb{E}_{\mathbb{Q}}\left[(k/B_1 - \bar{X}_1)_+\right], \quad k \in \mathbb{R}$$
$$p_2^A(k) = (k - x_0)_+ \vee \mathbb{E}_{\mathbb{Q}}\left[(k/B_1 - \bar{X}_1)_+ \vee \mathbb{E}_{\mathbb{Q}}[(k/B_2 - \bar{X}_2)_+ | \mathcal{F}_1]\right], \quad k \in \mathbb{R}$$

- p_1^A, p_2^A *potentials of* μ_1, μ_2 *centered in* x_0, *and* $\exists\, \nu_1, \nu_2$ *centered in* x_0 *such that:* $\nu_i|_{(-\infty, S(\mu_i)/B_1)} = B_1(G_\# \mu_i)|_{(-\infty, S(\mu_i)/B_1)}$, *with* $S(\mu) = \max \operatorname{supp}(\mu))$, $G(x) = x/B_1$, *and*

$$\nu_1 \prec_{s\beta_{12}} \nu_2, \qquad \text{where } \beta_{12} = 1 - B_1/B_2$$

Biased order and NA American prices

No stopping in 0, $\mathcal{F}_0$ trivial (cf. Cox-Hoeggerl 2024). Define the B_1-potential q of ν:

$$q(k) = \int (k/B_1 - x)_+ \, \nu(dx), \quad k \in \mathbb{R} \qquad \text{(i.e. } p(\cdot) := q(B_1 \cdot) \text{ is the potential of } \nu\text{)}$$

Theorem

For two maps $p_1^A, p_2^A : \mathbb{R} \to \mathbb{R}$, TFAE:

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- p_i^A B_1*-potential function of* ν_i, $i = 0, 1$, *such that*

$$\nu_1 \prec_{s\beta_{12}} \nu_2, \qquad \text{where } \beta_{12} = 1 - B_1/B_2$$

Models compatible with market prices

European t-Put $\forall k$ $\implies$ p_t^E B_t-potential of μ_t $\implies$ NA criteria:

$p_t^E(k)$ mk price $\mu_1 \prec_{\text{cvx}} \mu_2$

American t-Put $\forall k$ $\implies$ p_t^A B_1-potential of ν_t $\implies$ NA criteria:

$p_t^A(k)$ mk price $\nu_1 \prec_{s\beta_{12}} \nu_2$

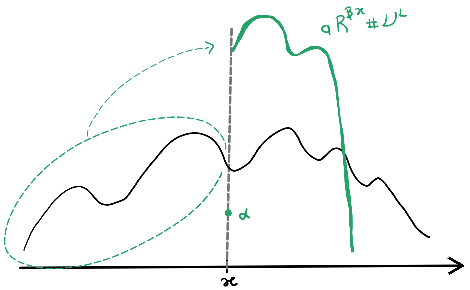
Characterization of β-biased measures

Theorem

A distribution $\nu \in \mathcal{P}_1(\mathbb{R})$ centered in x is β-biased if and only if

$$\nu^R \prec_{cvx} aR^{\beta,x}_{\#}\nu^L + \alpha\delta_x,$$

where $a = \frac{\beta}{1-\beta} > 0$, $\alpha = \|\nu^R\| - a\|\nu^L\| \geq 0$, and $R^{\beta,x}(y) = x - \frac{y-x}{a}$ distorted reflection



Characterization of strongly β-biased measures

Recall: (μ, ν) irreducible if $\mu \prec_{cvx} \nu$ and $\exists$ open I s.t. $\mu(I) = \nu(\bar{I}) = 1$ and $p_\mu < p_\nu$ in I

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Symmetric distributions:

- $\beta = 1/2 \Rightarrow a = 1$, so $R^{1/2,x}(y) = 2x - y$ is simply the reflection w.r.t. x
- Any ν symmetric w.r.t. its barycenter x has $\nu^R = R_\#^{1/2,x}\nu^L$ and $\alpha = 0$
- Then any symmetric distribution is $\frac{1}{2}$-biased, but not strongly $\frac{1}{2}$-biased

Outlook

- Ordering among measures from multiple price curves
- Case of both European and American options market data
- Continuous-time setting

Thank you for your attention!