Risk aversion of insider and dynamic asymmetric information.

Albina Danilova

London School of Economics

(based on joint work V. Lizhdvoy, HSE, Department of Mathematics)

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Review of Literature

- Kyle (1988), Back (1992), Back and Pedersen (1998), Wu (1999)

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- Single risky asset: dividend $V = \eta_{V(1)}$ where

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- $a \in C^{1,2}([0,1], I)$ is positive and satisfies

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  2. $V(t) - t$ converges to zero sufficiently slow.

Alternative representations of signal:

- via a time change $Z_t = \eta_{V(t)}$ and therefore:

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$\Rightarrow$ weak conditioning.

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$$\sup_{\theta \in \mathcal{A}(H)} \mathbb{E}^{c,z}\left[-e^{-\gamma W_1^\theta}\right] = \sup_{\theta \in \mathcal{A}(H)} \mathbb{E}^{c,z}\left[-e^{-\gamma\left[(V-P_1)\theta_1 + \int_0^1 \theta_s dP_s\right]}\right],$$

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  We will look for P satisfying

$$dP_t = w(t, P_t)dY_t^c + \frac{w_x(t,\xi_{t-})}{2} w(t,\xi_{t-})(d[Y,Y]_t^c - dt) + J_t, \tag{2}$$

  where $J_t = K_w^{-1}(t, K_w(t,\xi_{t-}) + \Delta Y_t) - \xi_{t-}$,
  $K_w(t,x) = \int_0^x \frac{1}{w(t,y)}dy + \int_0^t \frac{w_x(s,0)}{2}ds$.

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An admissible trading strategy $\theta \in \mathcal{A}(w)$ for insider satisfies:

1. θ is an $\mathcal{F}^{B,Z}$ adapted semi-martingale with summable jumps.
2. There exists a unique strong solution of (2) with $Y_t = B_t + \theta_t$.
3. (P, Z) is an $(\mathcal{F}_t)$ Markov process with measure $\mathbb{P}^{c,z}$.
4. $\mathbb{E}^{c,z}\left[e^{-\gamma \int_0^1 P_t dB_t - \frac{\gamma^2}{2}\int_0^1 P_t^2 dt}\right] = 1.$

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2. Insider optimality condition: given w^*, θ^* solves the insider optimization problem:

$$\mathbb{E}^{c^*,z}\left[-e^{-\gamma W_1^{\theta^*}}\right] = \sup_{\theta\in\mathcal{A}} \mathbb{E}^{c^*,z}\left[-e^{-\gamma W_1^{\theta}}\right].$$

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We focus on inconspicuous equilibrium, i.e the one with $\mathbb{E}[\theta_t|\mathcal{F}_t^Y] = 0$ for every $t \in [0, 1]$.

Theorem

Suppose the admissible pricing rule (w, c) satisfies $c = 0$ and

1. $$\frac{w_t(t,p)}{w^2(t,p)} + \frac{w_{pp}(t,p)}{2} = -\gamma.$$

2. θ^* *is admissible absolutely continuous strategy that satisfies*

$$P_1^* = Z_1, \; \mathbb{P}^{0,z} \; a.s..$$

In above P^* is the strong solution to

$$P_t = \int_0^t w(s, P_s) d(B_s + \theta_s^*).$$

Then θ^* is the optimal strategy.

Proof

Define function

$$\Psi^a(t,p) = \int_V^p \frac{y-a}{w(t,y)} dy + \frac{1}{2}\int_t^1 w(s,a) ds.$$

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Then

$$W_1^\theta = \Psi^{Z_1}(0,0) - \Psi^{Z_1}(1-,P_{1-}) - \frac{1}{2}\int_0^{1-} w(t,P_{t-})d[\theta,\theta]_t^c +$$

$$+ \sum_{0<t<1}\{\Psi^{Z_1}(t,P_t) - \Psi^{Z_1}(t,P_{t-}) - (P_t - Z_1)\Delta\theta_t\} +$$

$$+ \int_0^{1-}(P_t - Z_1)dB_t + \int_0^{1-}\frac{\gamma}{2}(P_{t-} - Z_1)^2 dt.$$

Proof, ctd.

Insider's utility is given by:

$$
\begin{aligned}
J &= -\frac{1}{\gamma}\inf_{\theta}\mathbb{E}^{0,z}\left[e^{-\gamma W_t^{\theta}}\right] \\
&\leq -\frac{1}{\gamma}\inf_{\theta}\mathbb{E}^{0,z}\left[e^{-\gamma\left(\Psi^{Z_1}(0,0)-\Psi^{Z_1}(1,P_1)\right)}\mathcal{E}_1(-\gamma(P-Z_1))\right] \\
&\leq -\frac{1}{\gamma}\mathbb{E}^{0,z}\left[e^{-\gamma\Psi^{Z_1}(0,0)}\right],
\end{aligned}
$$

where

$$
\mathcal{E}_t(X) = \exp\left\{\int_0^t X_s dB_s - \frac{1}{2}\int_0^t X_s^2 ds\right\}.
$$

Characterisation of Equilibrium

Theorem

A pair $((w^*, c^*), \theta^*)$ is an inconspicuous equilibrium if:

1. $c^* = 0$ *and* w^* *satisfies*

$$\frac{w_t^*(t,p)}{w^*(t,p)^2} + \frac{w_{pp}^*(t,p)}{2} = -\gamma, \tag{4}$$

2. $Y^* = B + \theta^*$ *is a standard Brownian motion in its own filtration,*
3. $P_1^* = Z_1$, $\mathbb{P}^{0,z}$ *a.s. where* P^* *is the strong solution to*

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- Admissible? Need $\mathbb{E}^{0,z}\left[e^{-\gamma\int_0^1 P_t dB_t - \frac{\gamma^2}{2}\int_0^1 P_t^2 dt}\right] = 1$

Consider $(U_t, R_t) = (v(V(t), Z_t), v(t, P_t))$. It solves

$$\begin{aligned} dU_t &= \sigma^2(t)\gamma\lambda(V(t), U_t)dt + \sigma(t)d\beta_t \\ dR_t &= \frac{U_t - R_t}{V(t) - t}dt + dB_t, \end{aligned}$$

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Theorem

The equilibrium is given by $c = 0$,

$$w(t,x) = a(t,x)$$

and

$$d\theta_t = \alpha_t dt, \qquad \alpha_t = w(t, P_t)\frac{\rho_x(t, P_t, V(t), Z_t)}{\rho(t, P_t, V(t), Z_t)}, \tag{6}$$

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where

$$a(t) = \frac{1}{\gamma t + C}, \; V(t) = \frac{1}{\frac{\gamma}{\gamma q + C} - \gamma \int_0^t \Sigma^2(s)ds} - \frac{C}{\gamma}$$

$$C = \frac{-\gamma(1+q) + \sqrt{\gamma^2(1+q)^2 - 4\left(q\gamma^2 - \frac{\gamma(1-q)}{\int_0^1 \Sigma^2(t)dt}\right)}}{2}$$

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- Choose $V(t)$ satisfying assumptions and a BM $\tilde{\beta}$.
- Construct

$$d\hat{\eta}_t = a(t, \tilde{\eta}_t) d\hat{\beta}_t + \frac{\rho_x(t, \hat{\eta}_t, 1, \eta_1)}{\rho(t, \hat{\eta}_t, 1, \eta_1)} dt$$

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- Shi (2015)

impose conditions ensuring existence of such a.

- Choose $V(t)$ satisfying assumptions and a BM $\tilde{\beta}$.
- Construct

$$d\hat{\eta}_t = a(t, \tilde{\eta}_t) d\hat{\beta}_t + \frac{\rho_x(t, \hat{\eta}_t, 1, \eta_1)}{\rho(t, \hat{\eta}_t, 1, \eta_1)} dt$$

- As $\hat{\eta}_1 \equiv \eta_1$, can use $\hat{\eta}$ as the dynamic signal, so equilibria given by $w(t, x) = a(t, x)$ and

$$d\theta_t = \alpha_t dt, \qquad \alpha_t = w(t, P_t) \frac{\rho_x(t, P_t, V(t), Z_t)}{\rho(t, P_t, V(t), Z_t)}, \qquad (7)$$

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Thank you