A joint framework for SPX, VIX and VXX

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This article introduces a novel framework that simultaneously accommodates three interconnected processes: the underlying S&P 500 index, the CBOE Volatility Index (VIX), and the iPath S&P 500 VIX Short-Term Futures ETN (VXX). Each of these assets possesses its own implied volatility surface in the options market, presenting a complex modeling challenge. The signature-based methodology employed in this study allows for a flexible and efficient representation of the joint dynamics, overcoming limitations of traditional modeling techniques. We first consider a stochastic volatility model in which the dynamics of the VIX index is modeled by a linear combination of the signature of an underlying polynomial process $\langle \ell, \widehat{\mathbb{X}}_t \rangle$. By exploiting the properties of the expected signature of a polynomial process, we derive the dynamics of the VXX index in closed form. Furthermore, using the analytical definition of the VIX index as the conditional expected value of the integrated volatility of the SPX asset over a 30-day time window, we are able to retrieve the dynamics of the volatility as a linear combination of the signature of the underlying polynomial process $\langle h(\ell), \widehat{\mathbb{X}}_t \rangle$. Leveraging the flexibility of signature methods, we propose a groundbreaking approach that links the dynamics of these three assets, resulting in a comprehensive model capable of calibration across all three volatility surfaces concurrently. Our results demonstrate a more coherent representation of the volatility term structure across all three assets.