Constrained deep learning for pricing and hedging european options in incomplete markets

Nicolas Baradel[*1]

[1]Ecole Polytechnique – Polytechnique - X – France

Abstract

In incomplete financial markets, pricing and hedging European options lack a unique no-arbitrage solution due to unhedgeable risks. This paper introduces a constrained deep learning approach to determine option prices and hedging strategies that minimize the Profit and Loss (P&L) distribution around zero. We employ a single neural network to represent the option price function, with its gradient serving as the hedging strategy, optimized via a loss function enforcing the self-financing portfolio condition. A key challenge arises from the non-smooth nature of option payoffs (e.g., vanilla calls are non-differentiable at-the-money, while digital options are discontinuous), which conflicts with the inherent smoothness of standard neural networks. To address this, we compare unconstrained networks against constrained architectures that explicitly embed the terminal payoff condition, drawing inspiration from PDE-solving techniques. Our framework assumes two tradable assets: the underlying and a liquid call option capturing volatility dynamics. Numerical experiments evaluate the method on simple options with varying non-smoothness, the exotic Equinox option, and scenarios with market jumps for robustness. Results demonstrate superior P&L distributions, highlighting the efficacy of constrained networks in handling realistic payoffs. This work advances machine learning applications in quantitative finance by integrating boundary constraints, offering a practical tool for pricing and hedging in incomplete markets.

[*]Speaker