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# The Guyon–Lekeufack Path-Dependent Volatility Model in Discrete Time: Calibration under $\mathbb{P}$ and $\mathbb{Q}$

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## Abstract

This talk presents different calibration approaches under both the risk-neutral measure  $\mathbb{Q}$  and the historical measure  $\mathbb{P}$  for a discrete-time version of the path-dependent volatility model introduced by Guyon and Lekeufack. We first illustrate the model's ability to fit option data across multiple dimensions, including VIX time series and joint SPX/VIX smiles. We then turn to the estimation of the model under the historical measure via maximum likelihood and show that the considered model outperforms existing approaches in the literature according to likelihood-based metrics. In addition, we highlight the overall congruence between the measure induced by the  $\mathbb{P}$ -estimated model and the average behavior of option market data. The results indicate, on the one hand, that the model captures well the joint formation of the  $\mathbb{P}$  and  $\mathbb{Q}$  measures and, on the other hand, that this relationship appears to be relatively stable over time. Building on these observations, we introduce new estimation approaches that combine  $\mathbb{P}$  and  $\mathbb{Q}$  time-series data to enhance calibration robustness. In particular, since time-series calibration of SPX/VIX volatility surfaces using Monte Carlo-based approaches becomes impractical when the number of dates is large, a deep calibration approach is introduced. Within this framework, the different estimation approaches are assessed through a trading game, using the model as a volatility arbitrage tool.

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