On the surjectivity of the conditional expectation given a real random variable

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Abstract

In this paper, we investigate the distributions of random couples (X, Y) with X real-valued such that any non-negative integrable random variable f (X) can be represented as a condi-

tional expectation, f (X) = E(g(Y)—X), for some non-negative measurable function g. It turns out

that this representation property is related to the smallness of the support of the conditional law of

X given Y , and in particular fails when this conditional law almost surely has a non-zero absolutely

continuous component with respect to the Lebesgue measure. We give a sufficient condition for

the representation property and check that it is also necessary under some additional assumptions

(for instance when X or Y are discrete). We also exhibit a rather involved example where the

representation property holds but the sufficient condition does not. Finally, we discuss a weakened

representation property where the non-negativity of g is relaxed. This study is motivated by the
calibration of time-discretized path-dependent volatility models to the implied volatility surface.

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