

---

# Revisiting contract theory with volatility control

Emma Hubert\*<sup>1</sup>

<sup>1</sup>U Paris Dauphine – Université Paris Dauphine - PSL – France

## Abstract

In this talk, we revisit the resolution of continuous-time principal–agent problems with drift and volatility control, originally addressed by Cvitanic, Possamai, and Touzi (2018) (1) through dynamic programming and second-order backward stochastic differential equations (2BSDEs), and develop new results in this framework.

We begin by introducing an alternative problem in which the principal is allowed to directly control the quadratic variation of the output process. On the one hand, the resolution of this contractible-volatility problem follows the classical methodology of Sannikov (2008) (2), thus relying on standard (first-order) BSDEs only. On the other hand, we demonstrate that the original contract form introduced in (1) allows the principal to achieve her contractible-volatility value, thereby ensuring both the optimality of this contract form and the equivalence between the original and the alternative problems. At the same time, this alternative approach reveals that the optimality of the original contract form implicitly relies on an additional duality assumption, which was not identified in (1). This observation motivates the construction of new families of contracts that remain optimal even when the duality assumption fails.

Altogether, this line of work both simplifies and strengthens the existing theory of continuous-time principal–agent problems with volatility control, and opens new directions for further extensions and applications in economics and finance.

Talk based on joint works with Alessandro Chiusolo, Dylan Possamai, and Nizar Touzi.

### References

- (1) J. Cvitanic, D. Possamai, and N. Touzi. Dynamic programming approach to principal–agent problems. *Finance and Stochastics*, 22(1):1–37, 2018.
- (2) Y. Sannikov. A continuous–time version of the principal–agent problem. *The Review of Economic Studies*, 75(3):957–984, 2008.

---

\*Speaker